[illegible]

[illegible]

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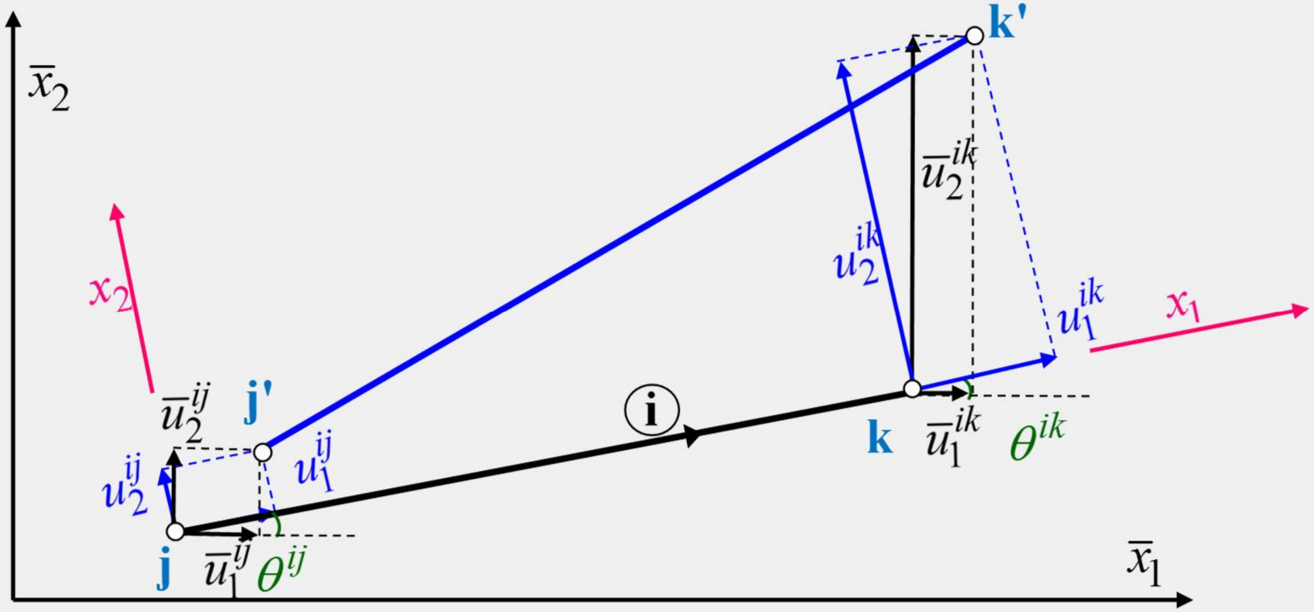
[illegible]



$$\left\{D^{ij}\right\} = \begin{Bmatrix} u_1^{ij} \\ u_2^{ij} \end{Bmatrix}$$

j

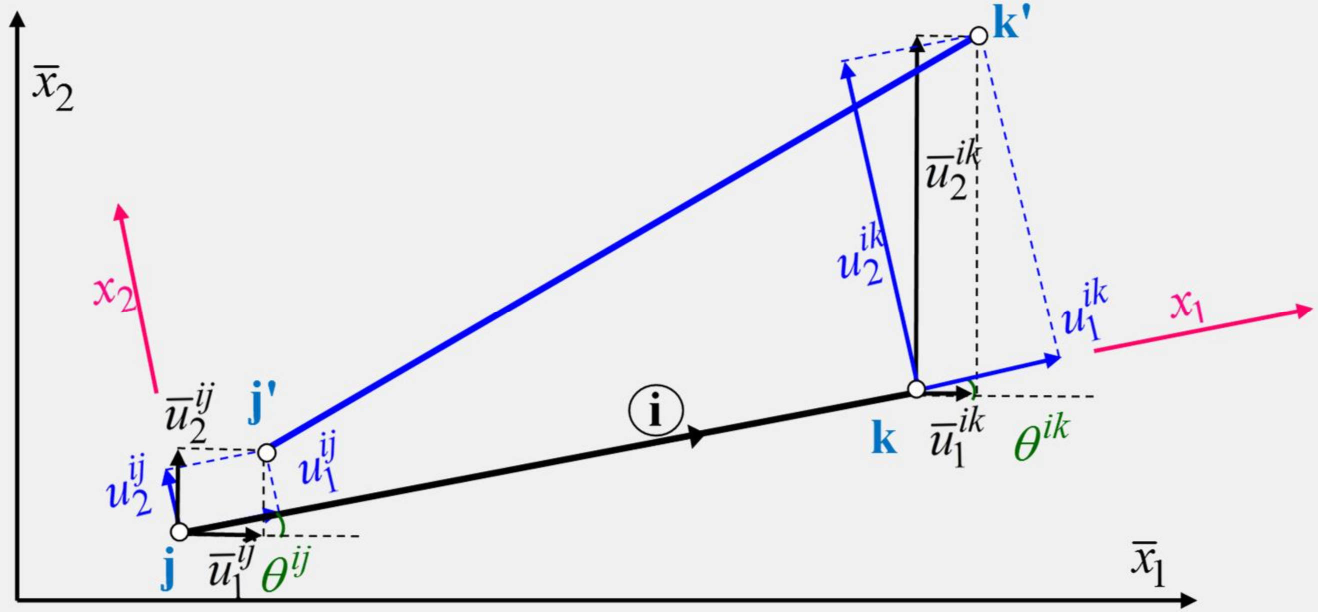
$$\left\{D^{ik}\right\} = \begin{Bmatrix} u_1^{ik} \\ u_2^{ik} \end{Bmatrix}$$

k

$$\left\{D^{i}\right\} = \begin{bmatrix} \left\{D^{ij}\right\} \\ \hline \left\{D^{ik}\right\} \end{bmatrix} = \begin{Bmatrix} u_1^{ij} \\ u_2^{ij} \\ \hline u_1^{ik} \\ u_2^{ik} \end{Bmatrix}$$

$$\left\{\bar{D}^{i}\right\} = \begin{bmatrix} \left\{\bar{D}^{ij}\right\} \\ \hline \left\{\bar{D}^{ik}\right\} \end{bmatrix} = \begin{Bmatrix} \bar{u}_1^{ij} \\ \bar{u}_2^{ij} \\ \hline \bar{u}_1^{ik} \\ \bar{u}_2^{ik} \end{Bmatrix}$$

[illegible]

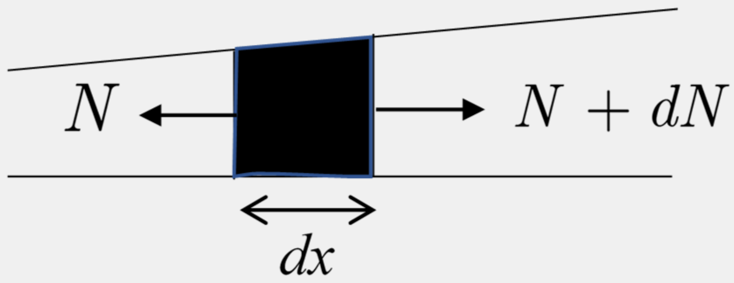


[illegible]: $u(x) = u_1^{ij}\psi_1(x) + u_1^{ik}\psi_3(x)$

$\psi_1(x), \psi_3(x) \rightarrow$ [illegible] $\rightarrow$ [illegible] $u_1^{ij} = 1, u_1^{ik} = 0$ [illegible] $u_1^{ij} = 0, u_1^{ik} = 1$, [illegible].

[illegible] ΩΜΑΤΟΣ

[illegible] dx [illegible]:



[illegible]

$$\frac{dN}{dx} = 0 \quad \text{[illegible]} \quad N = A\left(x\right)\sigma_x = EA\left(x\right)\varepsilon_x = EA\left(x\right)\frac{d\psi_i}{dx}$$

[illegible] $$\frac{d}{dx}\left[EA\left(x\right)\frac{d\psi_i}{dx}\right] = 0$$ [illegible] $$\psi_i = c_1\int_0^L \frac{1}{A\left(x\right)}dx + c_2$$

ɆɈʔȷ Λɇȼ ɆȺɫɆ Ɇɉ ȼʐȷ ɇɃɆ ɆɇɃɉ ȺɃɈ ȺɄɄȺɋɃɈ ɋɆɈɇɈɋʐȷ ɇɃɆ

ũɇŬ ɛɏəɞɠ Ů ά Ůɞɠ ŭəŰɡι ɛŬŰɞɠ
ůŰdŮɉ ɐɠ ŭɇŬŰɞɛɐɠ:

$$\psi_i\left(x\right) = c_1 x + c_2$$

ɃəůŰdŮɉ ɏɠ c_1 əŬə c_2 ɡ΄ ɞəɞɔəɞɜŰəŬ΄ ɧ Űɠ ůɡɜɞɉ ɇŬəɏɠ ůɡɜdɐəŮɠ.

ȮŰůɐ ɔɇŬ Ű̋ɞɜ ɣ $_1$(x): $\psi_1\left(0\right) = 1, \psi_1\left(L\right) = 0$

$$\boxed{\psi_1\left(x\right) = 1 - \xi,\ \xi = x/L}$$

Ůɞι , ɔɇŬ Ű̋ɞɜ ɣ $_3$(x): $\psi_3\left(0\right) = 0, \psi_3\left(L\right) = 1$

$$\boxed{\psi_3\left(x\right) = \xi,\ \xi = x/L}$$

[illegible]

[illegible]

[illegible]

$$\delta W_{\text{in}} = \int_V \sigma_x \delta\varepsilon_x dV = \int_0^L A(x) E \varepsilon_x \delta\varepsilon_x dx$$

V: [illegible]

$$\varepsilon_x = \frac{\partial u}{\partial x} = u_1^{ij}\psi_1'(x) + u_1^{ik}\psi_3'(x) = \begin{bmatrix}\psi_1'(x) & \psi_3'(x)\end{bmatrix}\begin{Bmatrix}u_1^{ij}\\ u_1^{ik}\end{Bmatrix} = \mathbf{N}'\mathbf{u}$$

$$\delta\varepsilon_x = \delta\frac{\partial u}{\partial x} = \delta u_1^{ij}\psi_1'(x) + \delta u_1^{ik}\psi_3'(x) = \begin{bmatrix}\psi_1'(x) & \psi_3'(x)\end{bmatrix}\begin{Bmatrix}\delta u_1^{ij}\\ \delta u_1^{ik}\end{Bmatrix} = \delta\mathbf{u}^T\mathbf{N}'^T$$

[illegible]

$$\mathbf{N} = \begin{bmatrix}\psi_1(x) & \psi_3(x)\end{bmatrix} = \begin{bmatrix}1-\xi & \xi\end{bmatrix}, \xi = x/L$$

[illegible] $\mathbf{u} = \begin{Bmatrix}u_1^{ij}\\ u_1^{ik}\end{Bmatrix}$

$$\mathbf{N}' = \begin{bmatrix}\psi_1'(x) & \psi_3'(x)\end{bmatrix}$$

[illegible]

[illegible]

$$\delta W_{\text{in}} = \delta \mathbf{u}^T \left(\int_0^L EA(x) \mathbf{N}'^T \mathbf{N}' dx \right) \mathbf{u}$$

[illegible]

$$\delta W_{ex} = \delta \mathbf{u}^T \mathbf{f} \quad \text{[illegible]} \quad \mathbf{f}^T = \begin{bmatrix} F_1^{ij} & F_1^{ik} \end{bmatrix}$$

[illegible] ($\delta W_{in} = \delta W_{ex}$):

$$\begin{Bmatrix} F_1^{ij} \\ F_1^{ik} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{13} \\ k_{31} & k_{33} \end{bmatrix} \begin{Bmatrix} u_1^{ij} \\ u_1^{ik} \end{Bmatrix} \quad \text{[illegible]} \quad \boxed{k_{ij} = \int_0^L EA(x)\, \psi_i'(x)\, \psi_j'(x)\, dx}$$

$$i, j = 1, 3$$

[illegible]

[illegible]

$$F_2^{ij} = F_2^{ik} = 0$$

[illegible]

$$\boxed{\begin{Bmatrix} F_1^{ij} \\ F_2^{ij} \\ F_1^{ik} \\ F_2^{ik} \end{Bmatrix}} = \begin{bmatrix} k_{11} & 0 & k_{13} & 0 \\ 0 & 0 & 0 & 0 \\ k_{31} & 0 & k_{33} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1^{ij} \\ u_2^{ij} \\ u_1^{ik} \\ u_2^{ik} \end{Bmatrix}$$

$$\left\{ A^i \right\}$$

[illegible]

[illegible]

$$F_2^{ij} = F_2^{ik} = 0$$

[illegible]

$$\underbrace{\begin{Bmatrix} F_1^{ij} \\ F_2^{ij} \\ F_1^{ik} \\ F_2^{ik} \end{Bmatrix}}_{\{A^i\}} = \underbrace{\boxed{\begin{bmatrix} k_{11} & 0 & k_{13} & 0 \\ 0 & 0 & 0 & 0 \\ k_{31} & 0 & k_{33} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}}_{\left[k^i\right]} \begin{Bmatrix} u_1^{ij} \\ u_2^{ij} \\ u_1^{ik} \\ u_2^{ik} \end{Bmatrix}$$

$$F_2^{ij} = F_2^{ik} = 0$$

$$\begin{Bmatrix} F_1^{ij} \\ F_2^{ij} \\ F_1^{ik} \\ F_2^{ik} \end{Bmatrix} = \begin{bmatrix} k_{11} & 0 & k_{13} & 0 \\ 0 & 0 & 0 & 0 \\ k_{31} & 0 & k_{33} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \boxed{\begin{Bmatrix} u_1^{ij} \\ u_2^{ij} \\ u_1^{ik} \\ u_2^{ik} \end{Bmatrix}}$$

$$\left\{ A^i \right\} \qquad \left[k^i \right] \qquad \left\{ D^i \right\}$$

$$\begin{Bmatrix} F_1^{ij} \\ F_2^{ij} \\ F_1^{ik} \\ F_2^{ik} \end{Bmatrix} = \begin{bmatrix} k_{11} & 0 & k_{13} & 0 \\ 0 & 0 & 0 & 0 \\ k_{31} & 0 & k_{33} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1^{ij} \\ u_2^{ij} \\ u_1^{ik} \\ u_2^{ik} \end{Bmatrix}$$

$$\left\{ A^i \right\} = \left[k^i \right] \left\{ D^i \right\}$$